

Prologue

Have you ever wondered how we evolved from blaming the gods for thunderstorms to predicting the weather with satellites, or casually checking the forecast on an app before leaving home?

For much of human history, humans turned to myths, deities, divine drama, and tradition to make sense of the world around them, but not by investigation. Thunder? Must be a deity throwing a tantrum. Disease? Probably a curse from an ancestor you forgot to pray to. An eclipse? Divine punishment. Although these stories may have brought comfort, they rarely solved real problems.

Enter science — the ultimate myth-buster. Science thrives on curiosity, evidence, and doubt; it doesn't settle for "because we've always done it this way". Unlike traditional explanations that often restricted progress, science opened the door to new discoveries. Science emerged not just as a method, but as a quiet revolution. It broke the monopoly of those who claimed to speak on behalf of the cosmos. It established that you don't need a priest or a prophet to understand the stars — you need a telescope. It gave us tools to challenge dogma, expose superstition, and dismantle systems built on unquestioned belief. It replaced fear with understanding, superstition with evidence, and 'meh' with 'hmm... Let's test that'.

At its core, science is about asking questions, testing ideas, and seeking proof. Instead of accepting things just because it's a belief, science encourages us to think critically and question everything. Where people once saw lightning as a divine tantrum, science explained it as an electrical discharge. This shift didn't just advance technology, it expanded freedom. The freedom to ask questions. The freedom to doubt. And what has that questioning spirit given us? Vaccines, smartphones, the Internet, Mars

rovers, and memes powered by WiFi. It has given us antibiotics without rituals, explanations without fear, and progress without permission.

Science — truth isn't what we're told — it's what we can prove

Science isn't just a school subject. It's the closest thing we have to an antidote to ignorance — and ignorance, history tells us, has often been enforced by those who feared the light of knowledge. Science is curiosity with a plan: A relentless quest to prove, not just to believe.

Physics: the Science of “How things actually work”

Physics is the branch of science that studies matter, energy, and the forces of nature—essentially, the user manual of the universe, explaining the reason apples fall, satellites orbit, and the cosmos doesn't collapse into chaos.

Science, in all its branches, is humanity's attempt to make sense of the world without resorting to divine guesswork. Biology tells us how living things function, chemistry explains how atoms and molecules interact, and geology uncovers the history written in rocks. Physics, however, goes deeper. It asks the more stubborn, fundamental questions: what matter is made of, how energy flows, and what invisible rules keep the universe running? Its reach spans mechanics, thermodynamics, electromagnetism, optics, and — eventually — quantum theory. Here, though, we begin at its most classical foundation: mechanics, the study of motion.

Think of physics as nature's source code, a way to peek behind the curtain and see what's really happening. Love WiFi? That's electromagnetic waves. Fancy MRI scans? Thank quantum mechanics. From black holes to bacteria, from bridges standing firm to rainbows lighting the sky — physics explains the “how” of it all.

Hot take: In a world full of noise — misinformation, fake news, viral conspiracies — physics is one of the last disciplines that insists on evidence. It doesn't care who's in charge, what the sacred text says, or how passionately someone argues. If it doesn't pass the test, it doesn't hold up.

Why it matters: Physics empowers you to think — not just believe. It trains your brain to ask bold questions and back them up with math and logic.

And remember: Physics isn't about replacing tradition. It's just not afraid to ask: "But what if tradition is wrong?" And when you have maths on your side, you don't need anyone's permission to find out.

Fun Fact: Physics is everywhere!

- Your TikTok went viral? Algorithms (okay, that's CS).
- Dropped your phone mid-scroll? That's gravity - Physics.
- Rainbows, rockets, rollercoasters? All physics.
- The fan above you? Rotational mechanics.
- The fridge humming in the kitchen? Thermodynamics. From mobile maps to delivery apps, from computer games to walking across the room — all of it obeys the laws of physics.

But don't take our word for it. Let's talk about something you use every single day without thinking twice.

Problem Statement: "How does my phone know where I am?"

Alright — so where does all this talk of questioning, proving, and thinking-for-yourself actually lead?

Let's look at something you use every day: Google Maps, Amap, or whatever location app helps you avoid getting lost. Ever wondered how your phone knows exactly where you are — sometimes better than you do? The answer lies not in magic or not in divine intuition. And no, there's not some invisible operator with a spy drone tracking your every move.

The answer lies in satellite navigation (Sat-Nav) systems like GPS (USA), Galileo (EU), BeiDou (China), or NavIC (India) [1]. They don't just 'guess' your location; they calculate it using signals from multiple satellites flying 20,000 km above your head, timing, and the laws of physics.

But here's the catch: to truly understand how this tech actually works, you don't need a PhD or prayers — you need Classical Mechanics. The same Newtonian laws that once explained why apples fall now help guide satellites, which in turn help your food delivery app. That's right – 17th-century physics is running your 21st-century tech.

Understanding this doesn't require blind belief. It requires vectors, motion, relative velocity, and yes... a dash of calculus. In a world where headlines shout and algorithms manipulate, physics quietly gets on with the job — calculating reality, one equation at a time.

So next time someone tells you science is just 'book stuff', remind them it's also the reason their map app doesn't send them into a lake.

Bottom line?

- We're about to reverse-engineer the tech that locates you on a map — using nothing but maths, logic, and good old Newtonian physics.
- Grab your critical brain, calculator, and maybe a snack – because physics is about to get seriously real.

Ready to uncover the physics behind the apps in your pocket? Let's dive in.

Satellite Navigation and the Role of Classical Physics

We ask our phones to do a lot — order food, send selfies, remind us about exams — but perhaps the most magical-seeming trick is this: they always know where we are. But how?

Satellite navigation systems (or Sat-Navs) determine your location by calculating how far electromagnetic signals, essentially light-speed pulses, have traveled from multiple satellites to your device. But to really grasp how this works, we'll need to unpack a few key questions:

- i. What exactly is a 'signal'?
- ii. How do satellites stay in orbit?
- iii. How does your phone deduce its own location just by receiving invisible pulses?

These aren't just technological queries; they're physics questions. And the answers lie in classical mechanics. Satellites orbit Earth at thousands of kilometers above us, constantly transmitting two things: (a) their precise location, and (b) the exact time the signal was sent.

Your phone receives these signals and measures the time it takes for each one to arrive. Since the signals travel at the speed of light, it can then estimate its distance from each satellite — all using a simple formula from basic kinematics: $\text{Distance} = \text{Speed} \times \text{Time}$. But there's more going on than just one signal and one distance. Your position is calculated through a process called *trilateration* (see Figure 1), which is where things get fun (and

SATELLITE NAVIGATION SYSTEM

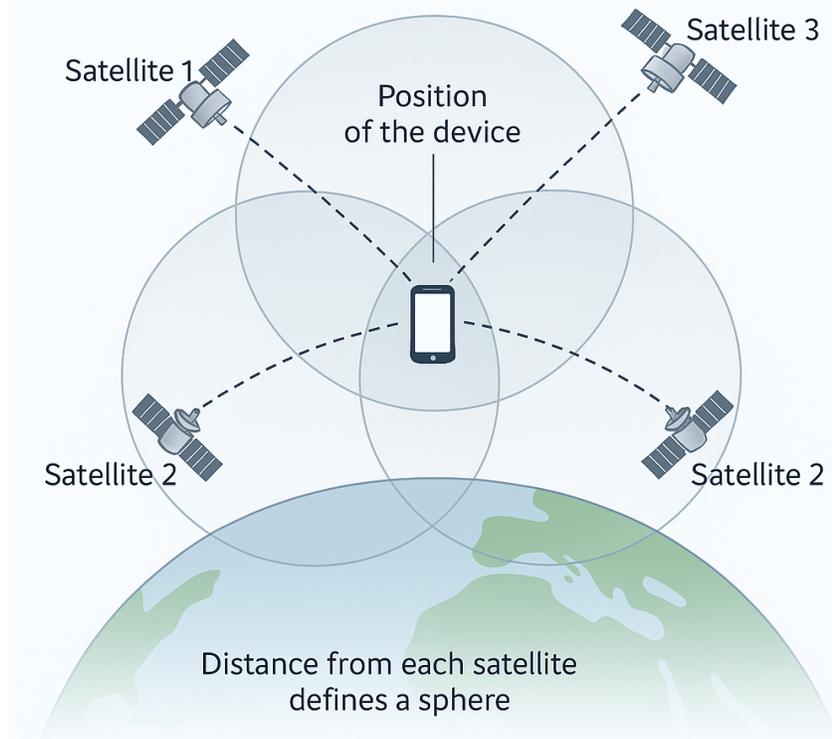


Figure 1: Modern-day birds of prey — satellites in orbit, silently watching and calculating.

technical). Here's a step-by-step breakdown:

- i. *Constellation of Satellites:* A network of satellites (usually 24-32) orbit Earth in predictable paths, constantly transmitting their position and the time of signal emission.
- ii. *Signal reception:* Your Sat-Nav-enabled device (like a phone or a car navigation system) must receive signals from at least four satellites to work out where it is.
- iii. *Time Delay Calculation:* Each signal, traveling at the speed of light, reaches your device at a slightly different time. The delay is tiny, just milliseconds — but it's enough to measure distance with surprising accuracy.
- iv. *Trilateration logic:* With one satellite, you know you're somewhere on a sphere around it. With two satellites, your posi-

tion is on the circle where the two spheres intersect. With three, it narrows down to two possible points. The fourth satellite helps eliminate the incorrect point and adjusts for the clock error in your device.

- v. *Final Location:* By combining distance data from four or more satellites, your device pinpoints your 3D position (latitude, longitude, altitude) and synchronizes its internal clock with Sat-Nav time.

From Curved Orbits to Straight Lines: Enter Classical Mechanics

- To fully understand how satellites move (and why they don't just fall), we begin with the simplest idea: motion in a straight line — or what we call one-dimensional (1D) motion. When an object moves along a straight path, we describe its position using displacement, velocity, and acceleration. These are not just numbers, but vectors, meaning they have both magnitude and direction.
- And as we build from motion along a line to two dimensions, we start to describe more complex cases: projectile motion, circular motion, and eventually orbital mechanics.

Think of a satellite as a projectile moving so fast that Earth's surface curves away beneath it. It's constantly 'falling', but never hits the ground. This beautiful balance of gravity and inertia, the push and pull, is explained through equations and concepts you'll soon learn. But none of it works without Newton.

So yes, the Sat-Nav system that helps you find the nearest Biriany stall is ultimately powered by the same man who watched apples fall.

Kinematics: Where Motion Meets Math

If you want to decode the tech that guides your Yatri-Sathi (mobile APP-based cab service in West Bengal) or your missile defense

system — it all begins with understanding how things move.

Before we dive into equations, orbits, and trajectories, let's take a moment to ask a deceptively simple question:

What does it mean for something to move?

Because if you can't figure out how to dodge a dodgeball, how will you ever colonize Mars?

Zeno's Paradox: Can You Ever Really Move?

The ancient Greek thinker Zeno of Elea posed a curious thought experiment. If you want to walk across a room, first you must go halfway. Then halfway through what's left. Then halfway again... and again... With infinite steps to take, shouldn't motion be impossible (see Figure 2)?

At first glance, it feels like a logical trap. If there are infinitely many points between here and your destination, how do you ever arrive? And yet — you just walked to this class from the tea-house!

Think!

Can you find the flaw in Zeno's reasoning? Hint: Think about infinite sums.

Spoiler: Calculus solved this! By summing infinite tiny steps (\int), we get finite results. So yes, you can reach class on time. (Homework: Use integrals to prove Zeno wrong. Bonus: Tag Zeno in a diss track.)

So, what's really going on here? To resolve paradoxes like this, and to understand any kind of motion, we need a precise mathematical language — the ideas of scalars, vectors, and eventually, calculus. To study how objects move, we need a way to describe where they are (position), how far they've gone (displacement), and how fast they're going (velocity). That's where *kinematics* begins.

Once we've got that sorted, we can define a position vector — a mathematical arrow that points from a chosen origin to an ob-

ject's location. So, displacement is the change in an object's position, and velocity is the rate and direction of that change. And to make sense of continuously changing quantities, like varying speed or acceleration, we'll need the basic ideas of calculus — specifically, derivatives. Because physics isn't just about knowing where things are — it's about understanding how they change.

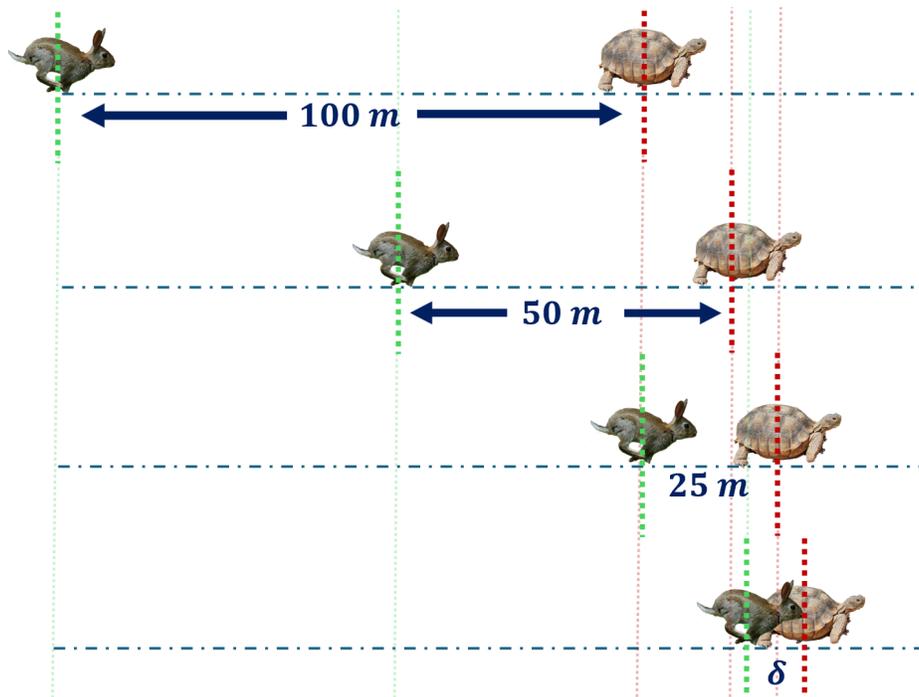


Figure 2: Zeno's other famous paradox — Achilles can never catch the tortoise, even though he's faster. Is it clever reasoning... or a misunderstanding of infinity?

So yes, to beat Zeno and build rockets, you'll need more than clever intuition. You'll need the language of mathematics and the discipline of physics. Let's build it, one step (or derivative) at a time.

Scalar vs Vectors

Math Toolbox

Don't Panic, It's Just Algebra's Edgy Cousin - Survival Kit for Physics.

Survival Skills: Vectors > scalars because direction matters IRL (e.g., running away from a bear).

Scalars: A quantity with only magnitude. Example: Temperature 30°C, Insta/youtube/rednote likes (1M+), Speed of car (50 m/s), Length of a track (100 m).

Vectors: A quantity with both magnitude and direction. Motion in more than one dimension requires vectors.

Example: Velocity (5 m/s north), Force (10 N down).

Why care?

If you're lost in a desert, "walk 5 km" (scalar) won't save you. "Walk 5 km east" (vector) might.

Vector operations:

In physics, vectors are fundamental tools for describing motion and forces. There are three key operations involving vectors:

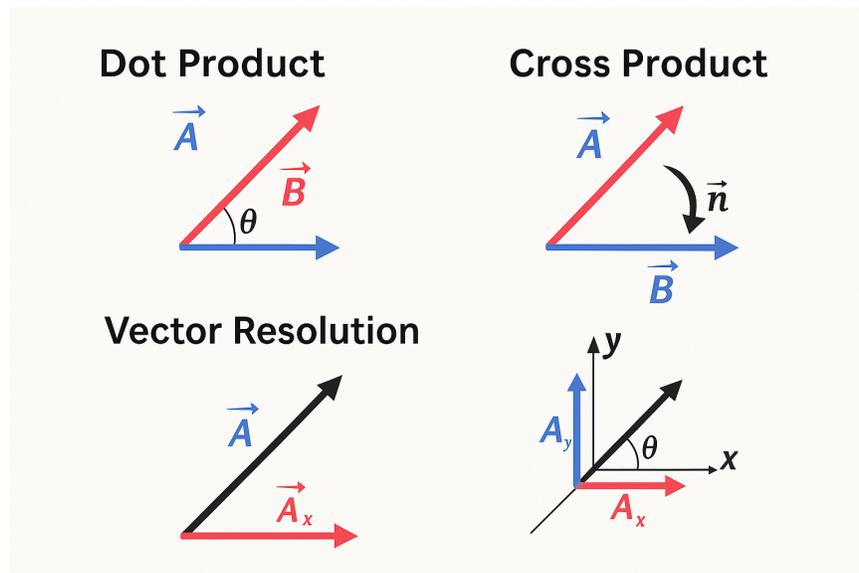


Figure 3: Basic vector operations.

(i) Addition, (ii) Dot operation (a.k.a. *Scalar product*), (iii) Cross operation (a.k.a. *Vector product*), and (iv) Vector Resolution into Perpendicular Components. Let's break them down one by one. One particularly important operation is vector resolution — breaking

a vector into components along perpendicular directions. This technique is extremely handy for simplifying problems and is central to the rules of vector addition.

The *dot product* is like a ‘compatibility test’ between two vectors — it tells you how much one vector influences another in a given direction. If two vectors are perpendicular, they have a zero dot product, meaning no influence on each other at all.

The *cross product*, on the other hand, is a little funkier. It creates a new vector that is perpendicular to both the original vectors. This weird-looking operation is crucial in analyzing rotation, torque, and angular momentum.

Representing a Vector Mathematically: Since a vector has both magnitude and direction, we represent it as

$$\vec{A} = A\hat{A}, \text{ where } A = |\vec{A}| \quad (1)$$

Here A denotes the magnitude (or length) of vector \vec{A} , and \hat{A} is a *unit vector*¹ pointing in its direction. By convention, we use \hat{x} , \hat{y} and \hat{z} to denote unit vectors along the Cartesian X -, Y -, Z -axes respectively.

To communicate the direction of a vector in a universally understandable way, we fix a reference coordinate system - typically, the three-dimensional Cartesian system defined by mutually perpendicular axes labeled X , Y , and Z . Once this coordinate system is fixed, any vector can be specified by describing its components along these three axes.

Importantly, it does not matter how one orients their coordinate system in physical space. Two people may draw their coordinate axes differently, but as long as the vector’s components are specified relative to each system, the vector itself remains objectively

¹Unit vectors have *unit magnitude* and are used to express direction only.

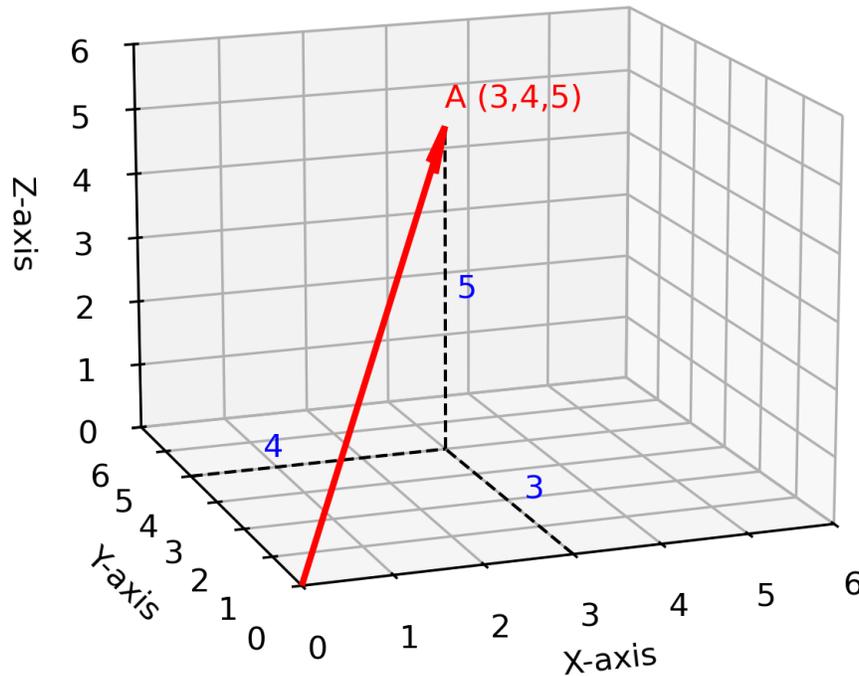


Figure 4: A vector represented in a 3D coordinate system.

the same. In essence, the description may vary, but the fundamental vector and its intrinsic physical significance remain unchanged regardless of coordinate transformations. Consequently, any conclusion derived using the vector within one coordinate system remains valid in another, provided the correct transformations are applied. Think of it like giving directions in different languages, the vocabulary changes, but the destination stays put.

This is one of the key advantages of mathematical language, it removes subjectivity introduced by natural languages like English or Bengali and enables the objective communication of ideas. For example, suppose someone wishes to draw a vector \vec{A} of magnitude $\sqrt{50}$. The direction of this vector can be specified by its components: 3 units along the X -axis, 4 along Y , and 5 along Z . These components can then be plotted in any coordinate system, and the resulting vector will be interpreted consistently across all observers (see Figures 4 and 5).

This naturally leads to the concept of the position vector, which

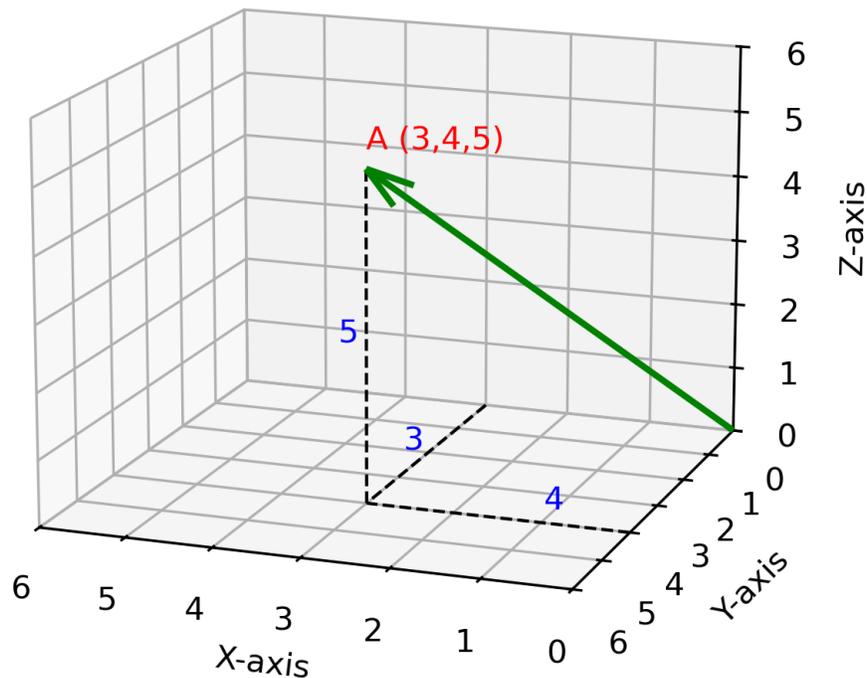


Figure 5: Rotating the axes doesn't change the vector — same arrow, different viewpoint.

represents the displacement from the origin to a point in space. It is a foundational notion in vector algebra and geometry.

Formally, consider two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in 3D-space. The vector from point A to point B , denoted as \vec{AB} , is given by

$$\vec{AB} = (x_2 - x_1)\hat{x} + (y_2 - y_1)\hat{y} + (z_2 - z_1)\hat{z} \quad (2)$$

The magnitude (or length) of this vector is

$$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad (3)$$

The reverse vector, i.e., the vector from B to A , denoted \vec{BA} , is simply the negative of \vec{AB} , $\vec{BA} = -\vec{AB}$. With this geometric understanding of vectors between two points, we are now in a position to define and understand vector addition.

Example: If you kick a football at an angle, it moves both horizontally and vertically. Resolving vectors helps us analyze this motion!

Addition

When two vectors are added, they follow the triangle (or parallelogram) rule: the resultant vector is the diagonal of the parallelogram formed by placing the two vectors tail-to-head.

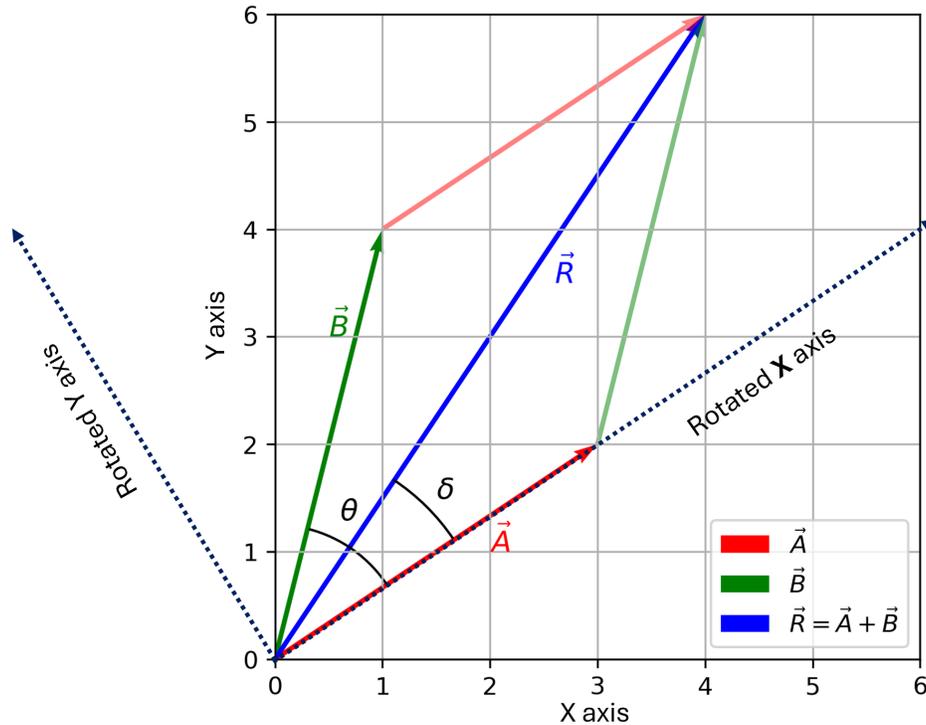


Figure 6: Adding vectors by building a parallelogram – the result (sum) is the diagonal pointing from the common starting point.

Let us now introduce the concept of vector addition using the previously discussed idea of vector representation, which essentially involves resolving a vector. Suppose we wish to determine the resultant of two vectors $\vec{A} = A\hat{A}$ and $\vec{B} = B\hat{B}$, that are separated by an angle θ (see Figure 6). Before we delve into the triangle or parallelogram rules of vector addition, let us apply the foundational concept we have just introduced, viz., that the first step in working with vectors is always to represent them in a coordinate system.

Note that we consider a $2D$ space for simplicity; however, the generalization to the $3D$ case is straightforward. In Figure 6, instead of adhering to the initial coordinate system, we may, without loss of generality, rotate the axes such that one of the vectors aligns

with either the X - or Y -axis. In this instance, we align the X -axis along \vec{A} . Consequently, in this rotated coordinate system, the two vectors are represented as follows

$$\vec{A} = A\hat{x}; \quad \vec{B} = B_x\hat{x} + B_y\hat{y} \quad (4)$$

Here, B_x and B_y are the components of vector \vec{B} along the two orthogonal directions. One can interpret these as perpendicular projections of \vec{B} onto the X - and Y -axes. Conceptually, if one wishes to move from the origin to the tip of \vec{B} , there are two equivalent paths: either by directly traversing along \vec{B} , or by taking B_x units along the X -axis followed by B_y units along the Y -axis.

Since \vec{B} makes an angle θ with the X -axis, elementary trigonometry gives $B_x = B \cos \theta$ and $B_y = B \sin \theta$. Therefore, in this rotated coordinate frame, \vec{B} can be expressed as

$$\vec{B} = B(\cos \theta \hat{x} + \sin \theta \hat{y}) \quad (5)$$

The resultant vector \vec{R} is then given by

$$\vec{R} = \vec{A} + \vec{B} = (A + B \cos \theta) \hat{x} + B \sin \theta \hat{y} \quad (6)$$

To compute the magnitude of \vec{R} , we consider its projections along the X - and Y - axes. Let \vec{R} subtend an angle δ with X -axis (as shown in Figure 6). Then we write,

$$R_x = R \cos \delta = A + B \cos \theta ; \quad R_y = R \sin \delta = B \sin \theta \quad (7)$$

Combining these, the magnitude and direction of \vec{R} are given by

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{A^2 + B^2 + 2AB \cos \theta}, \text{ with } \tan \delta = \frac{R_y}{R_x} = \frac{B \sin \theta}{A + B \cos \theta} \quad (8)$$

This result is particularly insightful, as it reveals that if two vectors are represented as adjacent sides of a parallelogram, then the diagonal originating from their common point corresponds to

their resultant. This is known as the parallelogram law of vector addition.

Equivalently, one may also describe vector addition using the triangle rule, where the sum of three vectors represented by the three sides of a triangle taken in a closed loop equals zero. As an exercise, the reader is encouraged to verify this by expressing the vectors in the $X - Y$ plane and computing their sum.

Dot product

The Ultimate Compatibility Test - Measures how much one vector aligns with another.

Translation: “How much do these vectors vibe in the same direction?”
 $\theta = 0 \implies$ Soulmates (max dot product). $\theta = \frac{\pi}{2} \implies$ “Let’s just be friends” (dot product = 0).

The dot product (also known as the *scalar product*) between two vectors is defined as

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta \quad (9)$$

The result of a dot product is a *scalar* quantity. It is widely used in: Work done by a force, $W = \vec{F} \cdot \vec{d}$ (Pushing a fridge sideways? Zero work. Physics approves laziness.); Finding the angle between two vectors (Tip: You might try this in a homework problem.).

Russell Street department

Cross product

When Vectors Create Drama - Measures how two vectors produce a perpendicular third vector.

Translation: “How strongly are these vectors trying to rotate around each other?” $\theta = 0 \implies$ Zero drama (vectors are aligned, cross product = 0).
 $\theta = \frac{\pi}{2} \implies$ Maximum drama (cross product is maximum).

The cross product (or *vector product*) between two vectors is de-

defined as

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin \theta \hat{n} \quad (10)$$

Here, the result is a vector (don't forget to notice \hat{n}), and this operation is also called the vector product. \hat{n} denotes a unit vector perpendicular to the plane formed by \vec{A} and \vec{B} . Determining the direction of \hat{n} is important, and it follows the right-hand-thumb rule - Align your fingers along the first vector (say, \vec{A}), then curl them towards the second (\vec{B}). Your thumb will point in the direction of \hat{n} . It might feel arbitrary, but it's a widely accepted and consistent convention, and surprisingly, it works! Used for: Torque (turning effect of a force), $\vec{\tau} = \vec{r} \times \vec{F}$. Loosening a bolt? Long wrench or short wrench (think). We will explore these ideas further when we study rotation and angular momentum in later chapters.

Helps in projectile motion (e.g., a basketball shot). Walking diagonally in Fortnite? Split velocity into $v_x = v \cos \theta$ and $v_y = v \sin \theta$. HW problem: Skateboarder slides down a 30° ramp at 10 m/s. Find v_x and v_y .

Basic Calculus: Why do we Need

Ever wondered why Newton invented calculus [2]? And yes, you could argue that Leibniz invented it first — or that Newton 'borrowed' it — but the truth is more interesting [3]. Both developed calculus independently, and for somewhat different reasons. Leibniz was tackling an optimization problem with roots in Archimedes' work, while Newton was focused on describing motion and change. But let's not get lost in that debate; historians can wrestle over priority disputes — we're just grateful it exists so physicists can use it!

The real reason calculus matters to us is motion. To describe how things change, how fast, in which direction, and by how much,

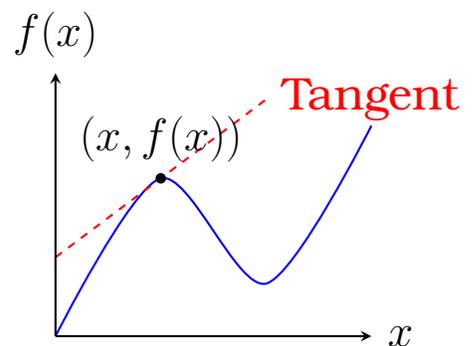
we need a mathematical language. That's precisely what calculus does. While a complete understanding of calculus encompasses concepts such as functions, limits, continuity, differentiation, and integration, this section provides an intuitive overview of differentiation and integration, along with their applications in the physical world. (For math enthusiasts, we recommend exploring the formal mathematical tutorial section [4] - an excellent resource for beginners.)

Differentiation:

Differentiation: Derivatives measure the rate of change (AKA “instantaneous slope”)

- Are your Insta followers growing? $\rightarrow f'(x) > 0$.
- Getting ratio'd on Twitter? $\rightarrow f'(x) < 0$.
- Want to predict where a BYD accelerating at $5m/s^2$ will be after $10s$?
- Curious how much rocket fuel you'll need to yeet Elon to Mars?

Geometric Intuition: Imagine you're driving a car and tracking your position over time. The resulting graph is curved. The derivative at any point tells you the slope of the tangent at that point, i.e., how steeply you are accelerating or braking in that instant.



Derivative = slope of the tangent;

Steeper slope \implies Rapid speed change

Formal definition: The derivative of a function $f(x)$ at a point is defined as

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (11)$$

This essentially means zooming in infinitely close until the curve appears to be a straight line. Then the slope of that line is the derivative.

Example: $f(x) = x^2$ - (A parabola), Derivative: $f'(x) = 2x$ (a straight line). At $x = 3$, the slope is $f'(x = 3) = 6$ - Very steep! You're zooming uphill. Knowing the slope helps us understand whether something is increasing, decreasing, or staying constant.

No Need to Memorise All Now

Basic Rules:

$$\frac{dc}{dx} = 0 \quad (12)$$

$$\frac{dx^n}{dx} = nx^{n-1} \quad (13)$$

$$\frac{d \sin x}{dx} = \cos x \quad (14)$$

Basic operations:

$$\frac{d(fg)}{dx} = f \frac{dg}{dx} + g \frac{df}{dx} \quad (15)$$

$$\frac{d(f + g)}{dx} = \frac{df}{dx} + \frac{dg}{dx} \quad (16)$$

$$\frac{df(g)}{dx} = \frac{df}{dg} \frac{dg}{dx} \quad (17)$$

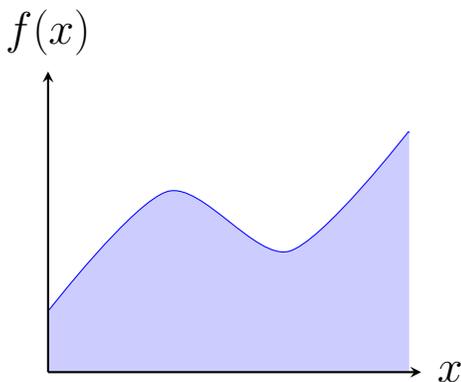
The flip side: understanding integration

If derivatives measure change, integration asks: how much total change has happened? Just as differentiation slices curves to understand instantaneous change, integration stitches those tiny changes back together to get the whole picture.

Integration: Area under the curve (AKA “accumulated quantity”)

Integration tells you how much total quantity has accumulated: distance, area, fuel used, electricity consumed.

- How much WiFi data did I use total this month?
- If you’re accelerating, how far did you go?
- Want to know how much rain fell on your roof over an hour?
- Got velocity as a function of time? Integrate it to get the distance.



$$\text{Total Area} = \int_a^b f(x) dx$$

Geometric Intuition: If differentiation is slope, integration is area under the curve. Think of a graph of speed vs time. The area under the curve gives the total distance traveled. Each small rectangle (tiny Δx slice) adds up to the total.

Total Accumulation = Area under the curve;

Formal Definition: The (definite) integral of a function $f(x)$ from a to b is defined as

$$\int_a^b f(x) dx = \lim_{\Delta x \rightarrow 0} \sum f(x_i) \Delta x \quad (18)$$

(Translation: Add up infinitely many thin rectangles to get total area.)

Example: For $f(x) = x^2$, the area under the curve between $x = 0$ and $x = 2$ is

$$\int_0^2 x^2 dx = \left[\frac{x^3}{3} \right]_0^2 = \frac{8}{3} \quad (19)$$

therefore, the quantity accumulated is $\frac{8}{3}$ units.

Don't Panic: Basic Integration Rules

Basic Rules:

$$\begin{aligned}\int c \, dx &= c x \\ \int x^n \, dx &= \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \\ \int \sin x \, dx &= -\cos x + C \\ \int \cos x \, dx &= \sin x + C\end{aligned}\tag{20}$$

Operations:

$$\begin{aligned}\int (f + g) \, dx &= \int f \, dx + \int g \, dx \\ \int c f \, dx &= c \int f \, dx\end{aligned}\tag{21}$$

You might be wondering, ‘Wait, where’s the operation that looks like $\int f(x)g(x), dx$?’ Well, buckle up, because that’s called integration by parts — and trust us, it’s the kind of thing that deserves its own episode (check your math tutorial for the sequel).

Here’s the deal: integration is basically the reverse Uno card of differentiation. For example, if $f'(x) = \frac{df}{dx}$, then you can go back by doing $f(x) = \int f'(x) \, dx$. Like, since $\frac{d}{dx} = \cos x$, that means $\int \cos x \, dx = \sin x$. Easy vibes.

But unlike differentiation, which has a universal definition (like the one given in Eq. 11), integration doesn’t always play fair. There’s no single, all-powerful definition that covers every situation. Some functions are just not integrable in terms of elementary functions. Yes, that’s right! Sometimes, it’s not you, it’s the function.

Final Thoughts

There's a famous piece of streetwise mathematical advice: 'If a stranger hands you a random function and asks you to integrate it, smile, nod, and walk away'. Because you might spend hours only to find it was never integrable to begin with. Stay safe out there.

Back to Kinematics

Position vector - "Where am I?"

Every moving object has a position at any instant, given by a position vector (Your Sat-Nav coordinates in vector form):

$$\vec{r}(t) = x(t) \hat{x} + y(t) \hat{y} + z(t) \hat{z} \quad (22)$$

Why Care?

Without it, you're lost. Physics (Coordinate system) needs to track where stuff is at all times. *This tells us exactly where the object is in space.*

Needless to say, this vector can change with time, because you're moving. Whether it's a car zooming on the highway or a planet orbiting the sun, its position vector $\vec{r}(t)$ evolves with time. This is the starting point of all kinematics.

Distance vs. Displacement: the "Steps vs. Straight line" feud

Distance (s, scalar): Total length of the path traveled between two points. Think: how many steps you actually took. The blue curved paths in Figure 7 represent the total length covered from point I to F.

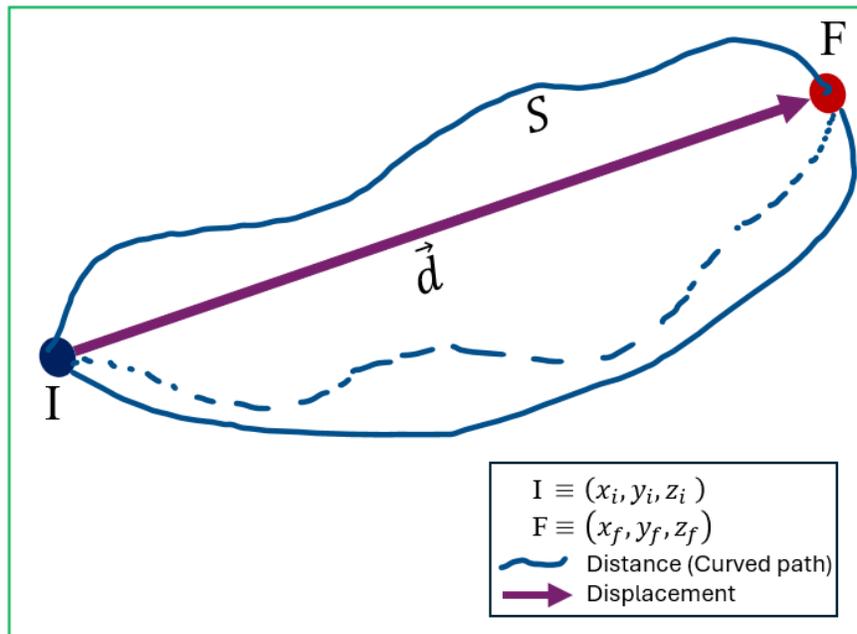


Figure 7: Took the scenic route? That's distance (blue curved path). Want the fastest way home? That's displacement (purple vector from I to F).

Displacement (\vec{d}): The shortest straight-line path from start to end, with direction included. Think: how far you are from where you began, not how tired your legs feel. Suppose you jog 5 km in a park but end up at the same point. The total distance is 5 km, but your displacement is 0!. You ended up exactly where you started. The purple directed straight line is the displacement from I to F. The displacement vector is given by the difference between the position vectors of F and I as

$$\vec{d} = (x_f - x_i) \hat{x} + (y_f - y_i) \hat{y} + (z_f - z_i) \hat{z} \quad (23)$$

Why Both?

- Distance tells you how much effort you burned (fitness apps care).
- Displacement tells you if you actually got somewhere (Yatri-Sathi drivers care).

Speed vs. Velocity: “How fast?” vs. “How fast + Where?”

Speed (v , scalar): Speed tells us how fast an object is moving; it is the rate of change of distance with respect to time. If a car’s position is described by a function $s(t)$, then its speed is given by [2]

$$v = \frac{ds(t)}{dt} \quad (24)$$

So, when speed changes, A car’s speedometer reads 60 km/h — this tells you how fast you’re going, but not where you’re headed.

Velocity (\vec{v} , vector): Velocity, on the other hand, tells us how fast and in what direction an object is moving. If a car’s position is described by a vector function $\vec{r}(t)$, then its velocity is

$$\vec{v} = \frac{d\vec{r}(t)}{dt} \quad (25)$$

Example: A car moving at 60 km/h north — here, both speed and direction matter. This is essentially what your Sat-Nav shows, not just the number on the dashboard.

Why Both?

- Speed: To avoid speeding tickets.
- Velocity: To land or launch rockets, flying planes (ISRO needs direction!).

To understand the difference more clearly, consider the example of uniform circular motion illustrated in Figure 8. “Uniform” here means the particle moves along a circular path of fixed radius r at constant speed. Suppose it takes time T to complete one full revolution. Then the speed is simply the circumference of the circle ($2\pi r$) divided by T .

However, even though the speed remains constant, the position vector $\vec{r}(t)$ keeps changing its direction at every instant — because

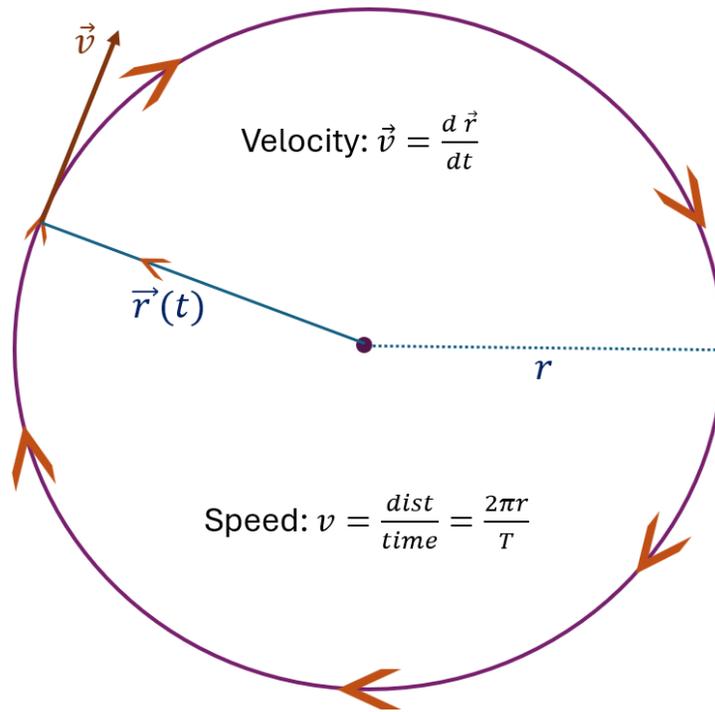


Figure 8: Going in circles at the same speed? Physics still says you're accelerating – because velocity isn't just 'how fast', it's 'which way'.

the particle is moving in a curve. As a result, the velocity (which is the time derivative of $\vec{r}(t)$) is constantly changing in direction, even though its magnitude (the speed) is unchanged.

So, in uniform circular motion: (i) Speed is constant, (ii) Velocity is not, because its direction keeps shifting. We'll explore circular motion in detail in later tutorials - stay tuned for that.

Acceleration: “Why motion feels like a Rollercoaster”

Acceleration tells us how fast the velocity changes – it is the rate of change of velocity, defined by

$$\vec{a} = \frac{d\vec{v}(t)}{dt} = |\vec{v}(t)| \frac{d\hat{v}(t)}{dt} + \frac{dv(t)}{dt} \hat{v}(t) = \frac{d^2\vec{r}(t)}{dt^2} \quad (26)$$

Here, $\vec{v}(t)$ is the velocity vector, $v(t) = |\vec{v}(t)|$ is the magnitude (speed), and $\hat{v}(t)$ is the unit direction vector of the velocity. Both speed and direction can vary with time. For instance, in non-uniform circular motion (such as a ceiling fan starting up), not

only does the speed change, but the direction also changes. Hence, both terms in the above equation contribute to acceleration.

Key Insight: Acceleration happens when you...

- i. Increase your speed, e.g., pressing the accelerator (\uparrow).
- ii. Decrease your speed, e.g., braking (\downarrow).
- iii. Change direction while moving, e.g., taking a turn ($\curvearrowright \rightleftharpoons \curvearrowleft$).

Important: Even if an object moves at constant speed in a circle, it still accelerates because the direction of velocity keeps changing!

Why these Three Concepts Rule Motion?

- i. Position: Pinpoints your location in space. (Where?)
- ii. Velocity: Tracks how fast and where you're headed. (How fast? + which way?)
- iii. Acceleration: Reveals if you're speeding, braking, or swerving. (Why is the motion changing?)

Important: Constant speed \neq zero acceleration. A car taking a turn at a steady speed still accelerates due to a change in direction!

Real-World Applications:

- I Sports Analytics: The Velocity of a football decides whether it scores.
Acceleration of a sprinter off the blocks wins races.
- II Car Safety: Airbags trigger upon rapid deceleration (i.e., high negative acceleration).
- III Space Exploration: Position vectors guide Mars rovers as they navigate alien terrain.
- IV Drone Delivery: **Position** - Where is it now? (Live Sat-Nav); **Velocity** - Is it coming toward you at 10 m/s? **Acceleration** - Did it suddenly decelerate to avoid a pigeon?

Deep Thought

Kinematics is the ABCs of motion. Master this, and you'll unlock dynamics (Newton's laws), energy and momentum, and even Einstein's relativity. Next stop: predicting the future with calculus! (See also Feynman's discussion of the power of mathematics in describing nature [5].)

Common Misconceptions

- i. Speed \neq Velocity: Saying "I walked 5 m/s" is speed. "5 m/s west" is velocity.
- ii. Distance \neq Displacement: Walking 1 km around the park \neq being 1 km from home.
- iii. Distance \neq $|\vec{Displacement}|$: specially in curved or non-linear motion, for instance, Drone Flying in Windy Conditions - The Sat-Nav path would show a longer route (actual path = distance covered). But the displacement - from initial GPS point to final GPS point - would be a much shorter straight-line distance.
- iv. Acceleration and velocity need not to be in same direction — In uniform circular motion, acceleration points towards the centre, while velocity is tangential.
- v. Zero Velocity \neq Zero Acceleration: A pendulum at its peak (turning point) has zero velocity but max acceleration! Throw a stone upward, at the turning point of the stone, the velocity as well as speed is zero, but acceleration is not (acceleration due to gravity is acting downwards).

Practice Problems: A dog runs 3 m east, then 4 m north in 5 seconds. Find: (a) Total distance travelled. (b) Displacement vector. (c) Average speed. (d) Average velocity.

Equations of motion: predicting the future of moving things

Motion means change in position over time. But how do we quantify this? That's where equations of motion come in. Think of

them as the physics equivalent of cheat codes for predicting how things move.

Key Reasons We Need Them:

- *Prediction:* Estimate where a Mars rover will land.
- *Analysis:* Figure out how your phone's screen shattered when dropped.
- *Design:* Engineer roller coasters that don't kill people.
- *Link to Forces:* Equations of motion describe what happens; Newton's laws explain why it happens (covered in an upcoming section).

Kinematics is all about describing motion mathematically. The basic quantities — displacement, velocity, and acceleration — are related through differentiation and integration.

From Position to Acceleration

$$\text{Position, } \vec{r}(t) \xrightarrow{\frac{d}{dt}} \text{Velocity, } \vec{v}(t) \xrightarrow{\frac{d}{dt}} \text{Acceleration, } \vec{a}(t) \quad (27)$$

Example: Consider a car climbing a mountain road that includes sharp U-shaped curves, known as hairpin bends, the kind you see in Darjeeling or the Western Ghats in India. The position vector of the car changes with time and can be modelled as

$$\vec{r}(t) = 20 \sin \frac{2\pi t}{5} \hat{x} + 10 t \hat{y} \quad (28)$$

Here, the sinusoidal x-component captures the zig-zag motion due to hairpin bends, while the linear y-component reflects steady elevation gain. This is depicted schematically in Fig.9.

The total distance travelled corresponds to the arc length of the actual curved path (shown in blue), which is longer than the straight-line displacement. The displacement vector, $\vec{FI} = \vec{r}$, connects the starting point (I) to the final point (F). The magnitude of this vector $|\vec{r}|$ (represented by the dashed-red straight line

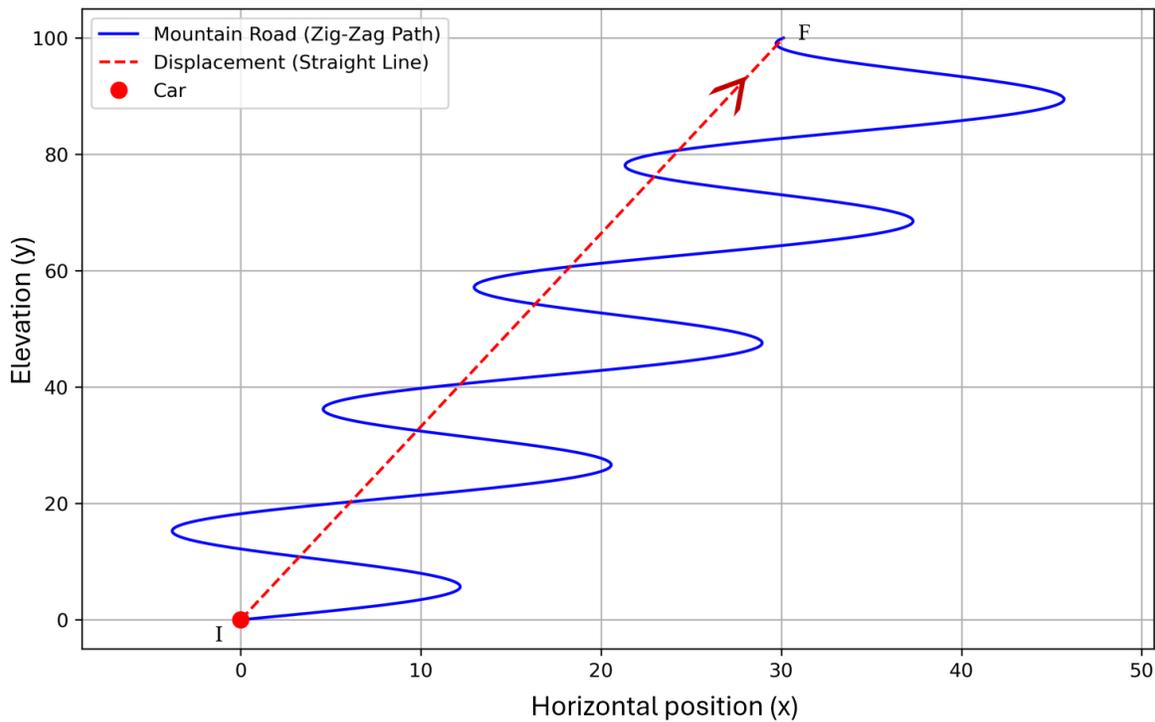


Figure 9: A Car Driving Up a Mountain via Hairpin Bends

IF), is clearly less than the total path length. The velocity and acceleration of the car are obtained by differentiating the position vector:

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = 8\pi \cos \frac{2\pi t}{5} \hat{x} + 10 \hat{y}; \quad \vec{a}(t) = \frac{d\vec{v}(t)}{dt} = -\frac{16\pi^2}{5} \sin \frac{2\pi t}{5} \hat{x} \quad (29)$$

From Acceleration to Position

$$\text{Acceleration, } \vec{a}(t) \xrightarrow{\int dt} \text{Velocity, } \vec{v}(t) \xrightarrow{\int dt} \text{Position, } \vec{r}(t) \quad (30)$$

Let's now switch to a simpler case: Let us consider a particle moving under constant acceleration along a straight path. (Small side note - saying "constant acceleration in a straight line" is a bit redundant, since constant acceleration automatically implies that the direction isn't changing, and hence the motion stays in one dimension. Still, you'll see this phrasing in many textbooks, so we're rolling with it.)

Assume the particle starts from point I (which we can conveniently choose as the origin, $(0, 0, 0)$) with initial velocity \vec{u} . Let's find its displacement and velocity after time t , when it reaches point F.

Solution: Given the acceleration, we apply Eq.(30) or Eq.(27), essentially the definition of acceleration—to determine the velocity and position. From the definition:

$$\vec{a} = \frac{d\vec{v}}{dt} \implies \int_{\vec{u}}^{\vec{v}} d\vec{v} = \int_0^t \vec{a} dt \implies \boxed{\vec{v} = \vec{u} + \vec{a}t} \quad (31)$$

If the initial velocity is $\vec{u} = u_x \hat{x} + u_y \hat{y} + u_z \hat{z}$, and the acceleration is constant along the x-direction, i.e., $\vec{a} = a\hat{x}$, then the y and z components of velocity remain unchanged. . Therefore, the final velocity becomes $\vec{v} = (u + at) \hat{x} + u_y \hat{y} + u_z \hat{z}$. This idea will come in handy when we study how Ronaldo bends a football in a free-kick, basically a 3D projectile motion situation, which we'll explore in the next tutorial.

Now that we've got the velocity, let's integrate once more to find the displacement

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} \implies \int_I^F d\vec{r} = \int_0^t (\vec{u} + \vec{a}t) dt \implies \boxed{\vec{r}(t) = \vec{u}t + \frac{1}{2} \vec{a}t^2} \quad (32)$$

Again, as there is no acceleration along the y- and z-axes, the magnitude of the displacements in these directions are simply: $y = u_y t$ and $z = u_z t$, so the total displacement vector is $\vec{r}(t) = (ut + \frac{1}{2} at^2) \hat{x} + u_y t \hat{y} + u_z t \hat{z}$. (For a rigorous treatment of these concepts, see [6].)

Homework Problems: Motion, Math, and the Real World

- i. **Dot Product Magic:** Calculate the following dot products using the standard unit vectors \hat{x} , \hat{y} , and \hat{z} — $\hat{x} \cdot \hat{x}$, $\hat{y} \cdot \hat{y}$, $\hat{x} \cdot \hat{z}$, and $\hat{z} \cdot \hat{y}$.
- ii. **Cross Product Power:** Calculate the following cross products using the standard unit vectors \hat{x} , \hat{y} , and \hat{z} — $\hat{x} \times \hat{x}$, $\hat{y} \times \hat{y}$, $\hat{x} \times \hat{z}$, $\hat{z} \times \hat{x}$, $\hat{x} \times \hat{y}$, and $\hat{z} \times \hat{y}$.
- iii. **Finding direction:** Given two vectors $\vec{A} = 3\hat{x} - 2\hat{y} + \hat{z}$ and $\vec{B} = 9\hat{x} + \hat{y} - 2\hat{z}$. Find the unit vectors perpendicular to the plane containing both \vec{A} and \vec{B} .
- iv. **Unlocking Vector Relationships:** Given two vectors $\vec{A} = -\hat{x} - 2\hat{y} + \hat{z}$ and $\vec{B} = 5\hat{x} + 2\hat{y} - \hat{z}$. Find the angle between them.
- v. **Morning Jog:** Riya jogs along a straight road for 2 km east, then 1.5 km west. (a) What is the total distance she covers? (b) What is her displacement? (c) Which of these quantities is scalar and which is vector? Explain.
- vi. **Hiker in the Hills:** A hiker moves 5 km in a direction 30° north of east, then 2 km due north. (a) Write both displacements in component form. (b) Find the resultant displacement vector.
- vii. **Food Delivery Challenge (Scalars, Vectors, and Basic Kinematics):** A delivery rider starts from the restaurant (point O) and drives 2 km due north in 4 min. She then turns 60° to the east and drives another 3 km in 6 min. (a) Represent each leg as a displacement vector (in components). (b) Find the resultant displacement vector and its magnitude. (c) Find her average speed and her average velocity (magnitude and direction). (d) If the rider had instead taken a straight road from O to the final position, keeping the same

average speed, how much time would she save?

- viii. **Metro + Escalator Conundrum:** You run at 2.5 m/s along a moving escalator of length 45 m. The escalator itself moves upward at 1.5 m/s. (a) Calculate the time taken to reach the top if you run up. (b) Now suppose you run down the same escalator (same running speed relative to escalator), find the time taken to reach the bottom.
- ix. **Following a Drone's Journey:** A drone is flying above a field. Its displacement from the starting point at time t seconds is given by the vector

$$\vec{r}(t) = 3t^2 \hat{x} + 4t \hat{y}, \quad (\text{in metres}) \quad (33)$$

- (a) Calculate the velocity, $\vec{v}(t)$ of the drone as a function of time. (b) Calculate the acceleration, $\vec{a}(t)$ of the drone as a function of time. (c) Find the speed of the drone at $t = 2$ seconds. (d) Calculate the total distance travelled by the drone from $t = 0$ to $t = 2$ seconds. (e) Calculate the magnitude of the displacement vector at $t = 2$ seconds. (f) Compare the magnitude of the displacement and the total distance travelled. Are they equal? Explain why or why not.

Outlook: from coordinates to curves of the sky

In this section, we began a journey through the unseen scaffolding that supports everyday technology, from taxi apps to missile guidance systems — all rooted in the timeless principles of classical mechanics. To appreciate how Sat-Nav systems determine your location with such precision, we laid the mathematical and conceptual foundations: scalars and vectors, position and displacement, speed and velocity. We explored how these quantities

interlink, and why calculus — the mathematics of change — is key to describing motion with precision.

This is just the beginning. To fully grasp how motion unfolds in our world, and how it is tracked and predicted by systems like Sat-Nav, we must now leave the comfort of straight lines and fixed coordinates. These include motion in curved paths, changing frames of reference, and relative perspectives — all of which are essential in both navigation and modern physics.

They shape everything from how pilots land planes to how astronomers measure starlight — and yes, how your phone keeps track of your location while you're weaving through traffic.

Coming up next

- **Reference frames:** Change your frame, transform the problem.
- **Relative velocity:** Because motion always depends on who's watching.
- **Projectile Motion:** Master this to predict anything thrown/shot/launched.
- **Circular Motion:** Where Newton's laws curve into orbits.

Stay curious. Your phone may know where you are, but true understanding begins when you know how it works.

References

- [1] C.-K. Hong, T.-S. Bae, and J. H. Kwon, "An overview of a special issue on upcoming positioning, navigation, and timing: Gps, glonass, galileo and beidou," *Remote Sensing*, vol. 14, no. 9, 2022, ISSN: 2072-4292. DOI: [10.3390/rs14091982](https://doi.org/10.3390/rs14091982). [Online]. Available: <https://www.mdpi.com/2072-4292/14/9/1982>.

- [2] I. Newton, *The Principia: Mathematical Principles of Natural Philosophy*. University of California Press, 1999, Translated by I. Bernard Cohen and Anne Whitman. Original work published 1687.
- [3] G. W. Leibniz, “Nova methodus pro maximis et minimis, itemque tangentibus, quae nec fractas nec irrationales quantitates moratur, et singulare pro illis calculi genus,” A new method for maxima and minima, and also for tangents, which is not impeded by fractional or irrational quantities, and a singular kind of calculus for them, Latin, *Acta Eruditorum*, pp. 467–473, Oct. 1684, English translation available in D. J. Struik (Ed.), *A Source Book in Mathematics, 1200–1800*, Harvard University Press, 1969, pp. 271–280. [Online]. Available: https://press.princeton.edu/books/hardcover/9780691638638/a-source-book-in-mathematics-1200-1800?srsltid=AfmB0or_tSJkYjhZxHIaUQFdTQATCLBLBsX0okaCoL0r8q9Qn5f2N8eZ.
- [4] S. Roy, “Tutorial on mathematics,” *Curiosita*, Jun. 2025. [Online]. Available: <https://curiositamagazine.com/math-tutorial/>.
- [5] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*. Addison-Wesley, 1964, vol. 1–3, Available online at <https://www.feynmanlectures.caltech.edu>.
- [6] D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics*, 11th ed. Hoboken, NJ: John Wiley & Sons, 2018, ISBN: 978-1-119-45701-9.