# Introduction

Imagine a blue ball and a red ball inside an opaque spherical shell. The shell is shaken up, so we have no idea the whereabouts of each of the balls inside the shell, and the balls are then shot out in opposite directions. Each of the balls is now caught, and we examine one of them. Let us say that we find it to be the red ball. We will have no doubt that when we examine the other ball, it will be the blue ball. We also would not doubt that the first ball had been red throughout the procedure and the second one had been blue. We certainly will not imagine that by our observation we 'made' the first ball red. And even more so, we will not imagine that by our observation of the first ball we 'made' the second ball blue. In fact, it will probably be surprising that we make such a fuss about such a simple and obvious 'experiment'.

It will then be surprising to learn that in quantum theory such discussions open up extremely interesting conceptual questions, which led to a much deeper understanding of the most fundamental ideas of physics. The two people who studied the implications of these arguments most deeply were Albert Einstein and John Bell. (See Parts 1 and 2).

# Prologue

John Bell made contributions to many areas of physics, including accelerators, quantum field theory, and the physics of elementary particles, but his most important work was the so-called Bell's inequality, which reopened the study of the fundamental nature of quantum theory and made important conceptual advances. There have been many applications, both theoretical and practical.

Of this article:

**Part 1** covers John Bell's early life and his first idea on quantum theory.

**Part 2** concentrates on his work on quantum theory and in particular his discovery of the Bell's Inequalities.

**Part 3** discusses a wide range of experiments, theoretical developments and conceptual ideas which have flown from Bell's work.

# Part I

## Family and Early Years

John Stewart Bell was born in Belfast in July 1928, the son of Annie and John, who was known as Jackie. To avoid confusion, John Stewart was called Stewart at home. He had an older sister, Ruby, and younger brothers, David and Robert.

Accounts of Bell's life nearly always suggest that the family was rather poor, and this very much annoyed Annie, who said that the children had everything that they have wanted, even if, as she said, it might, like bicycles, be second-hand – and of course much of the period when the children were growing up was wartime and there were no new bikes anyway!

In fact, the Bells and their standard of living were somewhat typical of most families in Belfast, and far further afield. It was the majority of physicists — for example, among those famous for their work on quantum theory, Bohr, Heisenberg, Schrödinger, Pauli, and Dirac — along with most other professional workers, who were the exception in coming from well-off households.

But John Bell and most of the 'typical' group did have at least one substantial step to climb — education.

Both Bell's parents were exceptionally hard-working, and Jackie undertook a wide range of tasks; he was prepared to turn his hand to anything. His family had been involved in horse dealing for many years, and Jackie continued this occupation, adding the sale of horse-drawn vehicles, which involved over 350 horses and 200 vehicles between 1924 and his enlistment in the army in 1940. Furthermore, he established a business as a fruiterer, employing two of his brothers. When he came back from the war with a stomach injury and a small pension, he took to running the British Legion car parks throughout Belfast, as well as organising squads of men to the North Coast to service the golf tournaments that took place there.

Annie became a private dressmaker, stitching 'night and day' for her customers.

John's parents also had extremely divergent views on education. His father had left school around the age of six to help his father with various tasks, and indeed, occasionally, they had had to pay a fine for his non-attendance at school. In contrast, though his mother was not particularly well-educated herself, she was exceptionally keen that her children should become as well-educated as possible. "Get educated," she said, "and you can wear your Sunday suit every day of the week."

Clearly, John Bell's parents were very different in several important ways, and it is fascinating to imagine how two such different people could have come together to create such an exceptional individual as John Bell.

# Education

As expected, John performed exceptionally well at primary school, and Annie would have very much liked him to proceed to the Belfast Royal Academical Institution (or Inst), probably the leading school in Belfast. He easily passed the exam to go to grammar school, but the catch was that education was not free in Northern Ireland at that time, and John would have had to be awarded a scholarship. However, when he took the scholarship exam for Inst, he was unsuccessful, and he had the same result at all the other grammar schools in Belfast. In retrospect, at least, this does seem shameful on the part of these schools, and it is concerning to realise how near the loss of such an outstanding scientist was.

Being ironic, however, we may realise that there must have been many higher status families to be satisfied, whose sons may have paid to attend the preparatory departments of these schools.

Figure 1: John Bell at school.

Fortunately, a small grant did at last materialize, by no means enough to pay for John to attend a grammar school, but sufficient for the Belfast Technical High School (or 'Tech'), a school of much lower status than Inst. Annie must have felt let down, but John studied most of the same subjects as at a grammar school. However, they also included bricklaying (theoretical only), carpentry (which he enjoyed), and bookkeeping (which he hated), rather than Latin or even Greek.

His father was away with the army around this time, so there was little argument about education, but he was back when Belfast suffered its worst period of bombing. He insisted that the family spend these few days with friends near the border with Southern Ireland.

Meanwhile, John became active in a wide range of hobbies: card games, chess, photography, stamp collecting, conjuring tricks, building radios, and he also became a vegetarian — exceptionally unusual for a young man without encouragement from anyone older. Obviously, he had a conscience!

He also became extremely interested in philosophy, drawn to figures such as Bertrand Russell, Bernard Shaw, and H.G. Wells, as well as the Brains Trust on the radio, but he eventually decided that philosophy was something of a dead end. One philosopher merely contradicted the previous one, and it was at this stage that he would concentrate on physics.

## Bell at Queen's University Belfast

By the age of 16, John had the qualifications to enter Queen's University in the city to study physics, which was his aspiration. However, money remained a concern. Additionally, he was a year too young to start at the university, so he needed to seek employment. He applied for several positions, including a junior role at the BBC, but he was unsuccessful, possibly because his true passion lay in physics.

In the end, he went with his mother to meet Professor Karl George Emeléus from the Physics Department at Queen's to inquire about any available jobs. He was offered a position as a junior laboratory assistant for the year, and although he thought at the time that this was just a standard role, it seems that Emeléus and his second-in-command, Robert Sloane, were so impressed with the young man that they created the position for him.

It was pretty basic, putting out apparatus and so on, but he was also assisted by Emeléus and Sloane in two ways. First, they lent him helpful books for study; more importantly, they allowed him to attend the first-year physics lectures. This helped him save a year in this part of the course, enabling him, after obtaining his first-class physics degree, to spend another year studying and earn a first-class math degree. There still remained the issue of funding. Fortunately, a senior laboratory assistant in the department had a connection with an organization that provided small grants for situations like this, and John was awarded such a grant. This support allowed him to successfully navigate his first year, and once he achieved outstanding performance, he received both a Queen's Second Foundation Scholarship and a Belfast Education Board Scholarship for the remaining years, making him relatively well-off for a student.

Figure 2: Registration at Queen's October 1945.

In his first year at Queen's, Bell engaged in extensive experimental work and acquired a significant amount of new mathematics, while his physics lectures primarily covered subjects he had previously encountered in school — mechanics, heat, light, electricity, and magnetism.

Then, in his second year of the course, the focus shifted to more theoretically based subjects: thermodynamics, statistical mechanics, quantum theory, relativity, and nuclear physics. While most of the courses were taught by Emeléus, Sloane delivered lectures on two subjects that had only gained prominence since the war: radio frequency atomic spectroscopy and nuclear accelerators.

While Bell found no difficulty with this new range of material — at least while attending the lectures, solving the problems, and passing the exams with top marks — he felt extremely uneasy about how quantum theory was presented by Emeléus, which was, in fact, similar to how just about every other lecturer in the world presented it.

It was indeed along the lines of: take the Schrödinger equation, insert the appropriate potential, solve the equation to obtain the energy levels and wave functions, use the difference between energy levels to create the spectrum, and so on.

This approach would certainly have satisfied nearly every student at universities worldwide (except for Bell?) and has indeed made a significant contribution to the vast amount of information gained from quantum theory and its applications over the past century. Of course, Bell would not have overlooked this — most of his own work focused precisely on this topic.

## Bell and Quantum Mechanics

But for him, it wasn't enough. Should quantum theory be such a virtually mechanical process? Should it not require meaning, a philosophy? Should it not tell us something about the physical world? What actually lies behind the equations?

> It is interesting that many of the problems studied by Bell in his career had been discovered by him while he was a student at Queen's.

Bell looked into this carefully, and he discovered that there were two approaches to these questions, both centered around the question of measurement, which gives us the only information we can obtain about the system. Studying this process may be regarded as penetrating the mathematical superstructure and attempting to find something definite about the system.

# Bell, Collapse and Hidden Variables

The first scheme is often referred to as that of John von Neumann, a great mathematician, though it is actually a fairly general approach [1]. To explain it, we will not use the most common case of wave functions in one dimension, such as for the simple harmonic oscillator, or in three dimensions, such as the hydrogen atom.

For simplicity, we will study electron spin. The electron possesses spin angular momentum of $\frac{\hbar}{2}$, where $\hbar$ is $\frac{h}{2\pi}$ and $h$ is Planck's constant. To be more precise, as we will see, we should state that measuring the z-component of the spin will yield a result of either $+\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$.

We may now write two states of the spin as $|+\rangle$ and $|-\rangle$ and, writing a measurement of the spin as $S_z$, we will have

$$S_z\,|+\rangle \to \frac{\hbar}{2}\,|+\rangle \qquad \text{and} \qquad S_z\,|-\rangle \to -\frac{\hbar}{2}\,|-\rangle \tag{1}$$

This seems quite simple, as we can measure either state and achieve the appropriate result. Additionally, the system remains in its original state.

However, the general case is much more complicated. $|+\rangle$ and $|-\rangle$, quantum mechanics includes states $a_1\,|+\rangle + a_2\,|-\rangle$ where $a_1$ and $a_2$ are numbers such that $a_1^2 + a_2^2 = 1$. So, examples might be

$\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ or, $\sqrt{\frac{1}{3}}\,|+\rangle + \sqrt{\frac{2}{3}}\,|-\rangle$.[1]

For this initial state, it is, of course, far from obvious what result we will obtain if we measure the z-component of spin. In fact, all we can state are probabilities:

The probability that we obtain the result $\frac{\hbar}{2}$ is $a_1^2$ and the probability that we obtain the value $-\frac{\hbar}{2}$ is $a_2^2$.

All that is essentially quantum theory, and the specific aspect of the von Neumann scheme is that if the result $+\frac{\hbar}{2}$ is obtained, then following the measurement, the state of the system collapses to $|+\rangle$, and if the result $-\frac{\hbar}{2}$ is obtained, then it collapses to $|-\rangle$. Therefore, if the measurement is immediately repeated, the same result will be obtained.

This scheme is called the collapse interpretation or the *projection postulate.*

An immediate problem with this interpretation is that the system behaves in a completely different manner when it evolves according to Schrödinger's equation compared to when it is measured and its state collapses. Von Neumann referred to these types of behavior as *Type 2* and *Type 1*, respectively.

While this is undoubtedly awkward, Bell particularly emphasized that it is also actually inconsistent. A measurement plays a special role in a physics experiment; however, as a Type 1 process, it is still composed of several Type 2 processes. How can its behavior be entirely different from a Type 2 process?

Another challenge with the von Neumann scheme is the fundamental rule of quantum theory that no information about a system exists beyond what is provided by the wavefunction (in this case, the state of spin).

[1] Technically we might note that $a_1$ and $a_2$ may be (mathematically) complex i.e. of the form $x + iy$ where $i = \sqrt{(-1)}$. Here, however, we ignore this complication and assume all these numbers are real.

According to standard quantum theory, no information exists about a system beyond what is provided by the wavefunction (state of spin in this case)

In this case, the system transitions from a single initial state to one of two final states, indicating a loss of determinism. Additionally, we must acknowledge that prior to the measurement, $S_z$ had no specific value at all, suggesting a loss of realism.

Both determinism and realism were central components of pre-quantum physics. In his ideas on quantum theory, Einstein was extremely unhappy about the loss of both. Bell was comparatively indifferent to the loss of determinism, but much of his future work on quantum theory would be based on trying to regain realism.

Figure 3: Albert Einstein. Source: [2]

In any case, the reader may be thinking, Bell certainly thought — why do we have to accept the restriction that there is no information not provided for us in the wavefunction? Could we not assume that even before the measurement, each system has a value of sz, and the measurement simply records this? This seems to solve the problem of the lack of determinism and realism rather trivially!

Technically, this extra variable would be called a hidden variable. However, since it is the value we obtain in a measurement, it is certainly not hidden! Of course, this is a very simple case and it would be accepted that other cases could be much more complicated. But we might still ask: Could we at least search for them?

However, Bell was disappointed to read that in 1932, von Neumann had claimed to have proved that no hidden variable theory could duplicate the predictions of quantum theory. The proof was included in a book published in German and was not translated at the time, so Bell had to accept it.

## Bell, Bohr and Copenhagen

The other approach to measurement was completely different — almost philosophical. It was nearly entirely the work of Niels Bohr. Since Bohr was Danish, this perspective is often referred to as the *Copenhagen Interpretation* of quantum theory, though not by its proponents — Heisenberg, Pauli, and others, who regarded it simply as quantum mechanics and believe that its correctness should not even be questioned. At that time, challenging these ideas meant disqualifying oneself from being considered a serious scientist.

For Bell, this was a pity, as he very much wanted to question it. Of course, he recognized that Bohr's earlier work on the Bohr atom, the atomic structure of the periodic table, the liquid drop model of the nucleus, nuclear fission, etc., established him as one of the greatest physicists of all time.

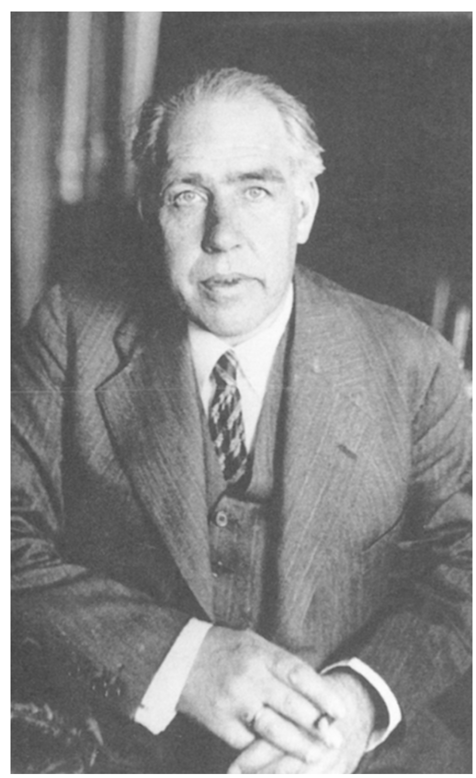

Figure 4: Niels Bohr.

Nevertheless, he had to admit that he found Bohr's interpretation of quantum theory totally unacceptable. Bohr discussed what he called the framework of complementarity, but

Bell preferred the word contradictory.

Like von Neumann, measurement was central for Bohr. He considered the classic example of the uncertainty principle, stressing that we should focus on actual physical measurements rather than limit ourselves to equations. He pointed out that when considering a measurement of momentum, one must think of the necessary apparatus, and the same applies to a measurement of position. However, it is impossible to set up equipment to measure both simultaneously, so one must not discuss the two quantities at the same time. This resolved the issue for Bohr, but not for Bell, who found the 'explanation' evasive and simplistic.

Bohr insisted that measurements and measurement systems must be treated classically, utilizing the positions of pointers on dials, marks on photographic plates, and so forth. Bell agreed.

But Bohr believed that the observed system itself should be described quantum mechanically. Here, Bell disagreed with him, thinking that it should be described in classical terms. This did not imply, of course, that it would obey classical laws; we know it must adhere to quantum laws. However, he argued that both observing and observed systems must share the same nature — they must both be real. Additionally, quantum and classical laws should be of the same general nature.

Here, Bell strongly disagreed with Bohr. According to Bohr, observing and observed systems were of different natures, so there must be a division between them, known as the *Heisenberg cut*. Bell preferred to call it the shifty split because Bohr acknowledged that due to the quantum nature of the observed system, the exchange of energy and momentum must be discontinuous and irregular, and we must consider one combined system. Bohr compared the situation to a walking stick. If we hold it tightly, we can view it as part of the observer, whereas if we hold it loosely, it is more reasonable to see it as part of the observed system.

Bell regarded these accounts as 'parables' and was extremely unimpressed with them.

## Bell and Peter Paul Ewald

In his fourth year, aside from one lecture course, all of Bell's time was devoted to a theoretical project. His supervisor was Peter Paul Ewald, who had just that year arrived at Queen's from Germany to lecture in mathematical physics.

Ewald was a physicist of the highest caliber. He played a significant role in collaboration with Max von Laue at the very beginning of X-ray crystallography, and his entire career was dedicated to that area of physics. By 1932, he was the Rector of the Technical University of Stuttgart, enjoying immense popularity and respect. However, being a quarter Jewish forced him to leave that position in 1933. He was allowed to teach until 1936, after which he had to seek a job outside Germany.

Figure 5: Peter Paul Ewald.

With so many others in the same position as he was, looking for jobs in the UK and USA, this was not easy; he ended up, as Bell put it, 'washed up on the shores of Ireland.' The position at Queen's was advertised as a lectureship, but during the interview, he was told that such a distinguished scientist as he would certainly be made a professor. However, almost immediately, there was a pronouncement that there would be no promotions for the duration of the war, and Ewart was not promoted until 1945!

Bell's project focused on the quantum mechanics of chain modules and was highly successful. Indeed, there was talk of it being published, although it seems that this did not occur.

Bell learned a great deal from Ewald, who also enjoyed interacting with him. Although Bell was concerned about advancing his career without a doctorate, Ewald assured him that this wouldn't matter as long as he 'had elbows.' Additionally, he was able to provide Bell with excellent references, which were especially valuable coming from such a respected physicist.

# Conclusion

By the end of his time as a student, Bell was well-positioned to embark on a successful career as a physicist, and he achieved this.

However, he felt disappointed that it seemed unlikely he would gain a deeper understanding of quantum theory. Fortunately, he was mistaken about this.

This will be discussed in Part II.

# References

[1] Travis Norsen. *Foundations of quantum mechanics*. en. 1st ed. Undergraduate Lecture Notes in Physics. To read online: https://www.google.com/books/edition/Foundations_of_Quantum_Mechanics/BaQxDwAAQBAJ?hl=en&gbpv=1&pg=PA1&printsec=frontcover. Cham, Switzerland: Springer International Publishing, Sept. 2017.

[2] Andrew Whitaker. *John Stewart Bell and Twentieth Century Physics*. en. London, England: Oxford University Press, Jan. 2020.